Commun. Math. Phys. 178, 83–105 (1996)

Communications in Mathematical Physics

Equipartition of the Eigenfunctions of Quantized Ergodic Maps on the Torus

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Received: 8 April 1995

Abstract: We give a simple proof of the equipartition of the eigenfunctions of a class of quantized ergodic area-preserving maps on the torus. Examples are the irrational translations, the skew translations, the hyperbolic automorphisms and some of their perturbations.

1. Introduction

Perhaps the simplest trace of the ergodicity of a Hamiltonian dynamical system one expects to find in the corresponding quantum system is the equipartition of its eigenfunctions in the classical limit. Such a phenomenon has been proved to occur in several cases. For the geodesic flow on compact Riemannian manifolds it is proved in [Sc, Z1, CdV]; for Hamiltonian flows on $\mathbb{R}^{2n}$ in [HMR] and for smooth convex two-dimensional ergodic billiards in [GL].

In this paper we study the quantization and the classical limit of certain area-preserving ergodic maps on the two-torus $T^{(2)}$, viewed as phase space, with canonical coordinates $(q,p) \in [0,a[\times[0,b[$. We will present a rather large class of models for which the desired equipartition result can be proved very easily. We will use the original idea of [Z1, CdV] which can be applied here with considerably less technical complications. Before doing so, we nevertheless first need to decide how to "quantize" an area-preserving map on the torus.

In Sect. 2 we describe the quantum Hilbert spaces associated to the torus. This problem has been addressed and solved by many authors before us with various different approaches [HB, BV, DE, DBDEG], always with the same result. The quantum Hilbert space is an N-dimensional complex vector space where N is related to $\hbar$ via the prequantum condition: $2\pi\hbar N = ab$. It carries an irreducible unitary representation of the discrete Weyl–Heisenberg group $\{(m\frac{a}{N}, n\frac{b}{N}, \phi) \in \mathbb{R}^3 \,|\, n, m \in \mathbb{Z}\}$. Here we give a rigorous version of the ideas of [HB] and [BV], which is

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