Commun. Math. Phys. 178, 61–82 (1996)

Communications in Mathematical Physics

On the Classification of Quantum Poincaré Groups

P. Podleś,[1]★,★★ S.L. Woronowicz[2]★★★

[1] Department of Mathematics, University of California, Berkeley CA 94720, USA
[2] Department of Mathematical Methods in Physics, Faculty of Physics, University of Warsaw, Hoża 74, 00-682 Warszawa, Poland

Received: 3 April 1995

Abstract: Using the general theory of [10], quantum Poincaré groups (without dilatations) are described and investigated. The description contains a set of numerical parameters which satisfy certain polynomial equations. For most cases we solve them and give the classification of quantum Poincaré groups. Each of them corresponds to exactly one quantum Minkowski space. The Poincaré series of these objects are the same as in the classical case. We also classify possible R-matrices for the fundamental representation of the group.

0. Introduction

The Minkowski space with the Poincaré group acting on it is the area of the quantum field theory. However, it is not known yet what is the area of a deeper theory, which would involve also the gravitational effects. It was suggested by many authors that it would be a quantum space. It means that instead of functions on spacetime we would have elements of some noncommutative algebra, called "the algebra of functions on the quantum space." On the other hand, such a quantum space should be in some sense similar to the ordinary Minkowski space. The simplest models of such a situation can be obtained by choosing some properties of Minkowski space endowed with the action of the Poincaré group and classifying all quantum groups and spaces which satisfy those properties. There are many examples of quantum Poincaré groups, the corresponding Minkowski spaces and R-matrices (cf. e.g. [4, 2, 11, 6, 5, 1, 15] and remarks in [10] concerning these papers) but such classification still doesn't exist. Our aim is to provide it. In Sect. 1 we define a quantum Poincaré group as a quantum group which is built from any quantum Lorentz group [14] and translations and satisfies some natural properties. The corresponding

★ On leave from Department of Mathematical Methods in Physics, Faculty of Physics, University of Warsaw, Hoża 74, 00-682 Warszawa, Poland.

★★ This research was supported in part by NSF grant DMS92-43893 and in part by Polish KBN grant No 2 P301 020 07.

★★★ This research was supported by Polish KBN grant No 2 P301 020 07.