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# Residue Formulas for the Large $k$ Asymptotics of Witten's Invariants of Seifert Manifolds. The Case of $SU(2)$

**L. Rozansky**[1]

Physics Department, University of Miami, P.O. Box 248046, Coral Gables, FL 33124, U.S.A.

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**Abstract:** We derive the large $k$ asymptotics of the surgery formula for $SU(2)$ Witten's invariants of general Seifert manifolds. The contributions of connected components of the moduli space of flat connections are identified. The contributions of irreducible connections are presented in the residue form. This allows us to express them in terms of intersection numbers on their moduli spaces.

## 1. Introduction

Let $A_\mu$ be a connection on an $SU(2)$ bundle $E$ over a 3-dimensional manifold $M$. The Chern–Simons action is a functional of this connection:

$$S_{\mathrm{CS}} = \frac{1}{2}\,\mathrm{Tr}\,\varepsilon^{\mu\nu\rho}\int_M d^3x\left(A_\mu\partial_\nu A_\rho + \frac{2}{3}A_\mu A_\nu A_\rho\right), \tag{1.1}$$

here Tr denotes a trace in the fundamental representation of $SU(2)$.

Consider an $n$-component link $\mathscr{L}$ in $M$. Let us attach $\alpha$-dimensonal irreducible representations $V_{\alpha_j}$ to the components $\mathscr{L}_j$ of $\mathscr{L}$. A partition function of the quantum Chern–Simons theory with the Planck constant

$$\hbar = \frac{2\pi}{k}, \quad k \in \mathbb{Z} \tag{1.2}$$

can be presented as a path integral taken with an appropriate measure over the gauge equivalence classes of $A_\mu$:

$$Z_{\{\alpha\}}(M,\mathscr{L};k) = \int[\mathscr{D}A_\mu]\,e^{\frac{i}{\hbar}S_{\mathrm{CS}}[A_\mu]}\prod_{j=1}^{n}\mathrm{Tr}_{\alpha_j}\,\mathrm{Pexp}\left(\oint_{\mathscr{L}_j}A_\mu dx^\mu\right), \tag{1.3}$$

here $\mathrm{Pexp}(\oint_{\mathscr{L}_j}A_\mu dx^\mu) \in SU(2)$ is a holonomy of $A_\mu$ along the contour $\mathscr{L}_j$ and $\mathrm{Tr}_\alpha$ is the trace in the $\alpha$-dimensional representation $V_\alpha$. We also use the following

[1] Address after September 25: L. Rozansky, School of Mathematics, Institute for Advanced Study, Princeton, N.J. 08540, USA.