Commun. Math. Phys. 178, 1 – 26 (1996)

Communications in
Mathematical
Physics

Yang–Mills–Higgs versus Connes–Lott

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Received: 3 April 1995

Abstract: By a suitable choice of variables we show that every Connes–Lott model is a Yang–Mills–Higgs model. The contrary is far from being true. Necessary conditions are given. Our analysis is pedestrian and illustrated by examples.

Despite its impressing success in describing particles and interactions, the Yang–Mills–Higgs (YMH) model building kit has conceptual shortcomings:

- its rules are essentially unmotivated,
- its complicated input comprising a Lie group and three representations is only justified by experiment,
- the model singled out by more and more precise experiments, namely the standard $SU(3) \times SU(2) \times U(1)$ model of electro-weak and strong interactions, is ugly and nobody really believes it to be the last word.

Concerning the first two points, the Connes–Lott (CL) model building kit [1] does better. Its rules have a precise motivation from non-commutative geometry and its input, comprising an involution algebra and two representations, is infinitely more restricted than the YMH input. Nevertheless, the standard model is also a CL model [1–4], a fact that by itself does not improve its beauty, but that perhaps allows unification with gravity. Indeed, the Einstein–Hilbert action as well may be formulated naturally in the setting of non-commutative geometry [5–7].

The purpose of this work is to show that the CL models represent a very small subset of the YMH models, where we restrict ourselves to "local" models, i.e. models defined on trivial bundles. Also we restrict ourselves to CL models defined by means of a finite dimensional algebra $\mathscr{A}$ tensorized with the algebra of functions on (a compact, Euclidean) "spacetime" of dimension 4. These particular models can be computed with elementary mathematics [8] and compare naturally to YMH models. Models whose algebras are not such tensor products, as the non-commutative torus [9], the fuzzy sphere [10] or a quantum space time [11] are

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