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Local Fluctuation of the Spectrum of a Multidimensional Anderson Tight Binding Model

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Abstract: We consider the Anderson tight binding model $H = -\Delta + V$ acting in $l^2(\mathbf{Z}^d)$ and its restriction H^Λ to finite hypercubes $\Lambda \subset \mathbf{Z}^d$. Here $V = \{V_x;\ x \in \mathbf{Z}^d\}$ is a random potential consisting of independent identically distributed random variables. Let $\{E_j(\Lambda)\}_j$ be the eigenvalues of H^Λ, and let $\xi_j(\Lambda, E) = |\Lambda|(E_j(\Lambda) - E)$, $j \geqq 1$, be its rescaled eigenvalues. Then assuming that the exponential decay of the fractional moment of the Green function holds for complex energies near E and that the density of states $n(E)$ exists at E, we shall prove that the random sequence $\{\xi_j(\Lambda, E)\}_j$, considered as a point process on $\mathbf{R}^1$, converges weakly to the stationary Poisson point process with intensity measure $n(E)dx$ as Λ gets large, thus extending the result of Molchanov proved for a one-dimensional continuum random Schrödinger operator. On the other hand, the exponential decay of the fractional moment of the Green function was established recently by Aizenman, Molchanov and Graf as a technical lemma for proving Anderson localization at large disorder or at extreme energy. Thus our result in this paper can be summarized as follows: near the energy E where Anderson localization is expected, there is no correlation between eigenvalues of H^Λ if Λ is large.

1. Introduction

In this paper, we treat the multi-dimensional Anderson tight binding model, namely the discretized Schrödinger operator H with a random potential V

$$H = -\Delta + V \tag{1.1}$$

acting in $l^2(\mathbf{Z}^d)$, where Δ is the discrete Laplacian defined by

$$(\Delta u)(x) = \sum_{|y-x|=1} u(y)\,. \tag{1.2}$$

We also consider the restriction H^Λ of H under the Dirichlet boundary condition to finite hypercubes $\Lambda \subset \mathbf{Z}^d$,

$$H^\Lambda = \chi_\Lambda H \chi_\Lambda\,, \tag{1.3}$$