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On the Quantum Group $SL_q(2)$

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Abstract: We start with the observation that the quantum group $SL_q(2)$, described in terms of the algebra of functions has a quantum subgroup, which is just a usual Cartan group. Based on this observation, we develop a general method of constructing quantum groups with similar property. We also develop this method in the language of quantized universal enveloping algebras, which is another common method of studying quantum groups. We carry out our method in detail for root systems of type $SL(2)$; as a byproduct, we find a new series of quantum groups – metaplectic groups of $SL(2)$-type. Representations of these groups can provide interesting examples of bimodule categories over monoidal category of representations of $SL_q(2)$.

Introduction

The goal of this paper is to analyze the notion of a quantum group. There are two approaches to this notion:

In the first approach, one describes a quantum group G in terms of a Hopf algebra $A = A(G)$ which plays the role of the algebra of functions on G. Then one studies the monoidal category of A-comodules, which is thought of as the category of representations of G. Our basic motivating example is the algebra A of regular functions on the quantum group $SL_q(2)$ (see [R-T-F, M]).

In the second approach, one describes the quantum group in terms of a Hopf algebra $U \subset A^*$, which plays the role of a universal enveloping algebra, and studies the tensor category of U-modules. This approach was initiated by Drinfeld [D] and Jimbo [J]. We use Lusztig's exposition (see [L]).

We decided to find a way from the first definition to the second one, by trying to understand what axioms and structures lie behind it.

We begin with the Hopf algebra $A = A_q$ of regular functions on $SL_q(2)$. This Hopf algebra supplies us with the material for axiomatic constructions and generalizations. Then our axiomatic approach gives us the direction in which to make further investigation of the Hopf algebra of regular functions on $SL_q(2)$, and so on.