Commun. Math. Phys. 177, 673–689 (1996)

Communications in Mathematical Physics


# Graded Poisson Lie Structures on Classical Complex Lie Groups

**G.E. Arutyunov**
Steklov Mathematical Institute, Vavilov 42, GSP-1, 117966, Moscow, Russia
E-mail: arut@qft.mian.su

Received: 21 November 1994 / Accepted: 11 September 1995

**Abstract:** The external algebra over holomorphic first order differential forms on a complex Lie group $G$ is endowed with the structure of a graded Poisson Lie algebra. This structure is introduced via graded bicovariant brackets that are shown to be in one to one correspondence with $G$-invariant tensors of special symmetry. Complete classification of graded Poisson Lie structures defined by homogeneous brackets is obtained for the case of classical complex Lie groups.

## 1. Introduction

The aim of this paper is to classify homogeneous Poisson Lie structures on the algebra of differential forms on a classical complex Lie group. This investigation is inspired by the issue of differential calculus on quantum groups for which the structures in question appear in the semiclassical approximation. Such a connection between graded Poisson Lie structures and differential calculus on quantum groups reflects the general Faddeev concept [1] that all objects in the quantum group theory should appear as the result of deformation (quantization) of appropriate Poisson structures [2–6].

Note that the notion of graded Poisson Lie structures [7] and the studies of their special examples [8–10] were stimulated by the papers [11, 12] where it was proved that bicovariant differential calculus on quantum groups can be supplied with the structure of a graded (super) Hopf algebra. At the semiclassical level this fact indicates the existence of graded Poisson Lie structures and the close analogy between such structures and Poisson Lie groups. Namely, a graded Poisson Lie structure governs the deformations of the external algebra in the same way as Poisson Lie group structures determine the deformation of the function algebra in the category of Hopf algebras. These deformed algebras can be viewed as algebras of external forms (functions) on a corresponding quantum group.