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Quantum Field Theories in All Dimensions

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Abstract: In this paper we exhibit a large class of hermitian scalar field theories satisfying the Wightman axioms. For each $d > 0$, and each polynomial P, we exhibit a collection of theories which are loosely but legitimately based on a $P(\phi)$ interaction in d space dimensions. One of the features of the construction is that the Wightman n-point function of each theory is a sum of finitely many integrals associated with "Feynman-like" graphs. Thus, it is in closed form.

0. Introduction

We present here a new approach to the construction of Wightman field theories. It needs a new idea to obtain nontrivial scattering, but there are several new elements in the proof of the Wightman axioms, of which we hope some may be useful in the construction of nontrivial theories.

Our theories are scalar field theories, based loosely but (we feel) genuinely on an arbitrary polynomial interaction in an arbitrary dimension. These theories satisfy the Wightman axioms, including vacuum uniqueness, and they are in some sense "perturbations" of a generalised free theory (see Definition 1.3).

However, they cannot be used directly to construct theories with nontrivial scattering amplitudes, firstly because the Wightman functions as given are "too smooth," a fact which cannot be remedied by merely adjusting things for fixed instead of "smoothly smeared" mass. Secondly, and more deeply, K.H.Rehren [Reh1] has succeeded in "decomposing" these theories into infinite Wick polnomials of countably many Gaussian fields with various smeared mass density functions (here an "infinite" polynomial means a polynomial in countably many variables with infinitely many nonzero coefficients but finite total degree, the coefficients being so chosen that the correlation functions converge in an appropriate manner). For the details of the "Rehren decomposition," one should of course consult [Reh1]; let us just remark that the decomposition is interesting in its own right, and by no means trivial. This result thus places our theories in the Borchers class of something we know to be trivial, and thus definitely precludes nontrivial scattering, at any rate in the case of fixed mass.