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Landau Hamiltonians With Random Potentials: Localization and the Density of States

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Abstract: We prove the existence of localized states at the edges of the bands for the two-dimensional Landau Hamiltonian with a random potential, of arbitrary disorder, provided that the magnetic field is sufficiently large. The corresponding eigenfunctions decay exponentially with the magnetic field and distance. We also prove that the integrated density of states is Lipschitz continuous away from the Landau energies. The proof relies on a Wegner estimate for the finite-area magnetic Hamiltonians with random potentials and exponential decay estimates for the finite-area Green's functions. The proof of the decay estimates for the Green's functions uses fundamental results from two-dimensional bond percolation theory.

1. Introduction

The existence of localized states for a two-dimensional gas of non-interacting electrons in a constant magnetic field is a main ingredient in various discussions and proofs of the integer quantum Hall effect (see e.g. [1–4, 7]). It is generally believed that localization occurs near the band edges for large magnetic fields and bounded, random potentials of arbitrary disorder. According to Halperin's argument [1], the localization length should diverge near the Landau levels. This is in contrast to the situation with no magnetic field. For two dimensional random systems, localization is expected to hold at all energies for arbitrary disorder and the eigenfunctions are expected to decay exponentially.

In this paper, we study the family H_ω of two-dimensional Landau Hamiltonians with Anderson-type potentials, having mean zero, on $L^2(\mathbb{R}^2)$. We prove that localization does occur in all energy intervals $I_n(B) \equiv [(2n+1)B + \mathcal{O}(B^{-1}), (2n+3)B - \mathcal{O}(B^{-1})]$, $n = 0, 1, 2, \ldots$ at large magnetic field strengths B and for arbitrary disorder. Recall that $\sigma(H_\omega)$ is contained in bands about the Landau levels

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