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Decay Rates of Solutions of an Anisotropic Inhomogeneous n-Dimensional Viscoelastic Equation with Polynomially Decaying Kernels

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Abstract: We consider the anisotropic and inhomogeneous viscoelastic equation and we prove that the first and second order energy decay polynomially as time goes to infinity when the relaxation function also decays polynomially to zero. That is, if the kernel G_{ijkl} satisfies

$$\dot{G}_{ijkl} \leqq -c_0 G_{ijkl}^{1+\frac{1}{p}}; \text{ and } G_{ijkl}, G_{ijkl}^{1+\frac{1}{p}} \in L^1(\mathbb{R}) \text{ for } p > 2 \text{ such that } 2^m - 1 < p ,$$

then the first and second order energy decay as $\frac{1}{(1+t)^q}$ with $q = 2^m - 1$.

1. Introduction

Several authors have studied the asymptotic stability of the solutions in viscoelasticity. Thanks to the works [1–5, 8, 9, 11] among others, it is well known that the stability holds for inhomogeneous and anisotropic n-dimensional materials and also for one-dimensional nonlinear equations. The question now is about the uniform rate of decay of the solution as time goes to infinity. Somehow, the way that the solution goes to zero depends on the decay of the kernel as time goes to infinity. We may ask, under what conditions on the kernel does the solution decay to zero exponentially or at least polynomially? To fix ideas, let us consider the simplest homogeneous isotropic n-dimensional viscoelastic equation with density $\rho = 1$,

$$u_{tt} - \mu \Delta u - (\mu + \lambda)\nabla \operatorname{div} u + \int_0^t g(t-\tau)[\mu \Delta u - (\mu + \lambda)\nabla \operatorname{div} u]\, d\tau = 0 , \qquad (1.1)$$

where λ and μ stand for Lame's constant and by g we denote the relaxation function. The kernel "g" plays an important role in the study of the asymptotic behaviour of the solutions.To see this, let us cite a few results about the uniform rate of decay. For example, in the work of Hrusa [8] the author showed, among others, properties

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