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Solitons and Vertex Operators in Twisted Affine Toda Field Theories

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Abstract: Affine Toda field theories in two dimensions constitute families of integrable, relativistically invariant field theories in correspondence with the affine Kac–Moody algebras. The particles which are the quantum excitations of the fields display interesting patterns in their masses and coupling which have recently been shown to extend to the classical soliton solutions arising when the couplings are imaginary. Here these results are extended from the untwisted to the twisted algebras. The new soliton solutions and their masses are found by a folding procedure which can be applied to the affine Kac–Moody algebras themselves to provide new insights into their structures. The relevant foldings are related to inner automorphisms of the associated finite dimensional Lie group which are calculated explicitly and related to what is known as the twisted Coxeter element. The fact that the twisted affine Kac–Moody algebras possess vertex operator constructions emerges naturally and is relevant to the soliton solutions.

1. Introduction

Affine Toda theories in two dimensions are integrable and possess an infinite number of local conversion laws [1, 2] whose charges generate what can be considered as an infinite dimensional Poincaré algebra,

$$[P^{(M)}, P^{(N)}] = 0\,, \tag{1.1a}$$

$$[K, P^{(M)}] = iMP^{(M)}\,. \tag{1.1b}$$

The Lorentz boost, K, measures the Lorentz spin, M, of the "momentum," $P^{(M)}$. The values of the integers M for which the momenta $P^{(M)}$ are non-zero form the set of exponents of the associated affine Kac Moody algebra whose root system appears in the original equations of motion. The affine Toda field theory possesses critical points with W-symmetry and the symmetries (1.1) can be regarded as the relics of this which survive when the critical theory is deformed in the appropriate integrable manner.