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Birkhoff–Kolmogorov–Arnold–Moser Tori in Convex Hamiltonian Systems

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Abstract: For the Hamiltonian systems of KAM type, it is proved that some lower dimensional invariant tori always exist in the resonance gaps although those maximum tori can not survive small perturbations in the generic case.

1. Introduction and Results

In this paper we study perturbed integrable Hamiltonian systems with n degrees of freedom and investigate what happens to those n-dimensional invariant tori of the unperturbed completely integrable systems in the zones of instability [A1].

To be more precise, the Hamiltonian system under consideration

$$\dot{q} = \frac{\partial H}{\partial p}, \qquad \dot{p} = -\frac{\partial H}{\partial q} \tag{1.1}$$

is determined by a Hamiltonian of KAM type

$$H(p,q) = N(p) + P(p,q) \tag{1.2}$$

which is assumed to be real analytical in $\mathbb{D} \times \mathbb{T}^n$, where $\mathbb{D} \subseteq \mathbb{R}^n$ is an open set, P is a small Hamiltonian perturbation and N is the main part. As usually, $q = (q_1, q_2, \ldots, q_n) \in \mathbb{T}^n$ denotes a vector of angular variables and $p = (p_1, p_2, \ldots, p_n) \in \mathbb{R}^n$ is a vector of action variables. Clearly, when the perturbation P vanishes, the system (1.1) is integrable and $\mathbb{D} \times \mathbb{T}^n$ is stratified by a family of n-dimensional invariant tori $p =$ const. carrying a quasi-periodic flow $q = \omega t + q_o$ with torus frequency vector ω given by

$$\omega(p) = (\omega_1, \omega_2, \ldots, \omega_n) = \frac{\partial N}{\partial p} \, . \tag{1.3}$$

If the frequency vector $\omega(p)$ is not too well approximated by rationals, in other words, if it satisfies some Diophantine condition, the famous KAM theory tells us that the corresponding invariant torus $p = p_o$ survives small perturbations with only