Commun. Math. Phys. 177, 469 – 506 (1996)

Communications in
Mathematical
Physics

A New Discrete Edwards Model and a New Polymer Measure in Two Dimensions

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Received: 31 March 1995

Abstract: The new discrete Edwards models in this paper are defined in terms of the so-called restricted intersection local times of the lattice random walk in two dimensions. We study the asymptotic behaviours of these new discrete Edwards models in the superrenormalizable cases. In particular, by approximating these models we can construct new polymer measures in two dimensions which are different from the original polymer measures obtained by approximating the original discrete Edwards models. The new discrete Edwards models can be thought of as zero-component lattice ϕ^4-fields with different cutoffs in the free and interacting parts.

1. Introduction

Let $\{B_t\}_{t \geqq 0}$ be the Brownian motion in R^d. The so-called polymer measure (or Edwards model) is formally defined by

$$v_\lambda(d\omega) = N_\lambda^{-1} \exp\left(-\lambda \int_0^1 \int_0^1 \delta(B_s - B_t) ds\, dt\right) \mu(d\omega), \tag{1.1}$$

where $\lambda \geqq 0$ is the coupling constant, N_λ is the normalization constant and μ is the Wiener measure. There has been a lot of works on the existence of the polymer measure v_λ. For instance, Varadhan (see Appendix to [28]) first proved the existence of v_λ for $d = 2$, and Stoll [26] then used the nonstandard approach to give a proof for the existence of the polymer measure v_λ for $d = 2$. For $d = 3$ and small enough $\lambda > 0$, Westwater [29] first constructed the polymer measure v_λ. At the same time as discussing the Borel summability, Westwater [30] proved that the polymer measure v_λ is also well defined for $d = 3$ and all $\lambda \in [0, \infty)$. Recently, Bolthausen [7] used an alternative approach with a simple proof, inspired by the approaches presented in [10] and [13], to construct the polymer measure v_λ for $d = 3$ and small enough $\lambda > 0$. In the following considerations we always assume that v_λ for $d = 3$ is the polymer measure defined by Bolthausen. For $d = 4$, it

★ Partly supported by AvH Foundation.