Character Expansion Methods for Matrix Models of Dually Weighted Graphs

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Abstract: We consider generalized one-matrix models in which external fields allow control over the coordination numbers on both the original and dual lattices. We rederive in a simple fashion a character expansion formula for these models originally due to Itzykson and Di Francesco, and then demonstrate how to take the large N limit of this expansion. The relationship to the usual matrix model resolvent is elucidated. Our methods give as a by-product an extremely simple derivation of the Migdal integral equation describing the large N limit of the Itzykson–Zuber formula. We illustrate and check our methods by analysing a number of models solvable by traditional means. We then proceed to solve a new model: a sum over planar graphs possessing even coordination numbers on both the original and the dual lattice. We conclude by formulating equations for the case of arbitrary sets of even, self-dual coupling constants. This opens the way for studying the deep problem of phase transitions from random to flat lattices.
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1. Introduction

After the considerable success of two dimensional quantum field theory and statistical mechanics – integrable models on 2D regular lattices, conformal field theories, Liouville theory and matrix models of 2D gravity and non-critical strings – progress in analytical results in this field has slowed down.

Among the principal questions remaining unsolved are, first, the so-called $c = 1$ barrier for non-critical strings (c is the central charge of the matter), and, second, the mysterious connection between the physical properties of various integrable 2D models coupled and non-coupled to gravity. The first problem is usually attributed to the absence of a stable vacuum for $c > 1$, though it has never been clearly

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