Commun. Math. Phys. 177, 417–434 (1996)

Communications in Mathematical Physics

Intersection-Equivalence of Brownian Paths and Certain Branching Processes

Yuval Peres

Department of Statistics, University of California, Berkeley, CA 94120, USA
Email: peres@stat.berkeley.edu

Received: 11 January 1995

Abstract: We show that sample paths of Brownian motion (and other stable processes) intersect the same sets as certain random Cantor sets constructed by a branching process. With this approach, the classical result that two independent Brownian paths in four dimensions do not intersect reduces to the dying out of a critical branching process, and estimates due to Lawler (1982) for the long-range intersection probability of several random walk paths, reduce to Kolmogorov's 1938 law for the lifetime of a critical branching process. Extensions to random walks with long jumps and applications to Hausdorff dimension are also derived.

1. Introduction

Random walk and percolation problems in regular trees (sometimes called "Bethe lattices") are well known to be easier than the corresponding problems in Euclidean space. In this paper we show that long-range intersection probabilities for random walks, Brownian motion paths and Wiener sausages in Euclidean space, can be estimated up to constant factors by survival probabilities of branching processes and percolation processes on trees. The following "dictionary" illustrates the reduction.

Problem in Euclidean space	**Corresponding problem on trees**
• How many (independent) Brownian paths in $\mathbf{R}^d$ can intersect?	• Which branching processes can have an infinite line of descent?
• What is the probability that several random walk paths, started at random in a cube of side-length 2^k, will intersect?	• What is the probability that a branching process survives for at least k generations?

Research partially supported by NSF grant # DMS-9404391 and a Junior Faculty Fellowship from the Regents of the University of California