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Spectral Analysis and the Haar Functional on the Quantum *SU* (2) Group

H.T. Koelink[1], J. Verding[2]

[1] Vakgroep Wiskunde, Universiteit van Amsterdam, Plantage Muidergracht 24, NL-1018 TV Amsterdam, The Netherlands. E-mail: koelink@fwi.uva.nl

[2] Departement Wiskunde, Katholieke Universiteit Leuven, Celestijnenlaan 200 B, B-3001 Leuven (Heverlee), Belgium. E-mail: Jan.Verding@wis.KULeuven.ac.be

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Abstract: The Haar functional on the quantum $SU(2)$ group is the analogue of invariant integration on the group $SU(2)$. If restricted to a subalgebra generated by a self-adjoint element the Haar functional can be expressed as an integral with a continuous measure or with a discrete measure or by a combination of both. These results by Woronowicz and Koornwinder have been proved by using the corepresentation theory of the quantum $SU(2)$ group and Schur's orthogonality relations for matrix elements of irreducible unitary corepresentations. These results are proved here by using a spectral analysis of the generator of the subalgebra. The spectral measures can be described in terms of the orthogonality measures of orthogonal polynomials by using the theory of Jacobi matrices.

1. Introduction

The existence of the Haar measure for locally compact groups is a cornerstone in harmonic analysis. The situation for general quantum groups is not (yet) so nice, but for compact matrix quantum groups Woronowicz [22, Thm. 4.2] has proved that a suitable analogue of the Haar measure exists. This analogue of the Haar measure is a state on a C^*-algebra. In particular, the analogue of the Haar measure on the deformed C^*-algebra $A_q(SU(2))$ of continuous functions on the group $SU(2)$ is explicitly known. This Haar functional plays an important role in the harmonic analysis on the quantum $SU(2)$ group. For instance, the corepresentations of the C^*-algebra are similar to the representations of the Lie group $SU(2)$, and the matrix elements of the corepresentations can be expressed in terms of the little q-Jacobi polynomials, cf. [14, 17, 20], and the orthogonality relations for the little q-Jacobi polynomials are equivalent to the Schur orthogonality relations on the C^*-algebra $A_q(SU(2))$ involving the Haar functional. This was the start of a fruitful connection

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