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Integrable Structure of Conformal Field Theory, Quantum KdV Theory and Thermodynamic Bethe Ansatz

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Abstract: We construct the quantum versions of the monodromy matrices of KdV theory. The traces of these quantum monodromy matrices, which will be called as "**T**-operators," act in highest weight Virasoro modules. The **T**-operators depend on the spectral parameter λ and their expansion around $\lambda = \infty$ generates an infinite set of commuting Hamiltonians of the quantum KdV system. The **T**-operators can be viewed as the continuous field theory versions of the commuting transfer-matrices of integrable lattice theory. In particular, we show that for the values $c = 1 - 3\frac{(2n+1)^2}{2n+3}$, $n = 1,2,3\ldots$ of the Virasoro central charge the eigenvalues of the **T**-operators satisfy a closed system of functional equations sufficient for determining the spectrum. For the ground-state eigenvalue these functional equations are equivalent to those of the massless Thermodynamic Bethe Ansatz for the minimal conformal field theory $\mathscr{M}_{2,2n+3}$; in general they provide a way to generalize the technique of the Thermodynamic Bethe Ansatz to the excited states. We discuss a generalization of our approach to the cases of massive field theories obtained by perturbing these Conformal Field Theories with the operator $\Phi_{1,3}$. The relation of these **T**-operators to the boundary states is also briefly described.

The studies of the last decade revealed a deep relation between the structures of Conformal Field Theory (CFT) [1], Integrable Field Theory [2,3] and Solvable Lattice Models [4]. The conformal symmetry of CFT is generated by its energy-momentum tensor $T(u)$, whose mode expansion

$$T(u) = -\frac{c}{24} + \sum_{-\infty}^{+\infty} L_{-n}\, e^{inu} \tag{1}$$

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