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Localized Endomorphisms of the Chiral Ising Model

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Abstract: Based on the treatment of the chiral Ising model by Mack and Schomerus, we present examples of localized endomorphisms ϱ_1^{loc} and $\varrho_{1/2}^{\mathrm{loc}}$. It is shown that they lead to the same superselection sectors as the global ones in the sense that unitary equivalence $\pi_0 \circ \varrho_1^{\mathrm{loc}} \cong \pi_1$ and $\pi_0 \circ \varrho_{1/2}^{\mathrm{loc}} \cong \pi_{1/2}$ holds. Araki's formalism of the selfdual CAR algebra is used for the proof. We prove local normality and extend representations and localized endomorphisms to a global algebra of observables which is generated by local von Neumann algebras on the punctured circle. In this framework, we manifestly prove fusion rules and derive statistics operators.

1. Introduction

In local quantum field theory one considers a Hilbert space $\mathscr{H}$ of physical states which decomposes into orthogonal subspaces $\mathscr{H}_J$ (superselection sectors) so that observables do not make transitions between the sectors. The subspaces $\mathscr{H}_J$ carry inequivalent, irreducible representations of the observable algebra $\mathscr{A}$, possibly with some multiplicities [19]. Among the superselection sectors, there is a distinguished sector $\mathscr{H}_0$ which contains the vacuum vector $|\Omega_0\rangle$ and carries the vacuum representation π_0.

The starting point in the algebraic approach to quantum field theory is the observable algebra $\mathscr{A}$ which is usually defined as the C^*-inductive limit of the net of local von Neumann algebras $\{\mathscr{A}(\mathscr{O}), \mathscr{O} \in \mathscr{K}\}$, where $\mathscr{K}$ denotes the set of open double cones in D dimensional Minkowski space. The net is assumed to satisfy the Haag-Kastler-axioms. In general, the observable algebra $\mathscr{A}$ admits a lot of inequivalent representations. Therefore one has to find an appropriate selection criterion which rules out the physically non-relevant representations. Doplicher, Haag and Roberts [10, 11, 18] developed the theory of locally generated sectors; they suggested that one has to consider only those representations π_J which become equivalent to the vacuum representation in the restriction to the causal complement $\mathscr{O}'$ of any sufficiently large double cone $\mathscr{O} \in \mathscr{K}$. That means that for a representation π_J satisfying the DHR criterion, there exists for each sufficiently large double cone $\mathscr{O}$ a unitary $V : \mathscr{H}_0 \to \mathscr{H}_J$ such that