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Renormalizability Proof for QED Based on Flow Equations

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Abstract: We prove the perturbative renormalizability of Euclidean QED_4 using flow equations, i.e. with the aid of the Wilson renormalization group adapted to perturbation theory. As compared to Φ_4^4 the additional difficulty to overcome is that the regularization violates gauge invariance. We prove that there exists a class of renormalization conditions such that the renormalized Green functions satisfy the QED Ward identities and such that they are infrared finite at nonexceptional momenta. We give bounds on the singular behaviour at exceptional momenta (due to the massless photon) and comment on the adaptation to the case when the fermions are also massless.

1. Introduction

About twenty years ago Wilson and his collaborators published their ideas on the renormalization group and effective Lagrangians [1], which have stimulated the progress of quantum field theory and statistical mechanics ever since. In 1984 Polchinski [2] showed that these ideas are suited for a treatment of the renormalization problem of perturbative field theory which does not make any use of Feynman diagrams and in particular sidesteps the complicated analysis of the divergence/convergence properties of the general bare or renormalized Feynman diagram. Instead he showed that the problem can be solved by bounding the solutions of a system of first order differential equations, the flow equations, which are a reduction of the Wilson flow equations to their perturbative content.

The present paper is part of a programme of the authors with the aim to show that the Polchinski method is suited to prove (in the sense of mathematical physics) the perturbative renormalizability of any (by naive power counting) renormalizable theory of physical interest. Polchinski's original proof for Euclidean massive Φ_4^4 was restricted to unphysical renormalization conditions (because they were imposed on the Green functions with an additional (large) infrared cutoff), and it was achieved

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