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Yang–Mills and Dirac Fields in a Bag, Constraints and Reduction

Jędrzej Śniatycki[1],*, Günter Schwarz[2],, Larry Bates[1],*****

[1] Department of Mathematics and Statistics, University of Calgary, Calgary, Alberta, Canada
[2] Department of Mathematics and Computer Science, University of Mannheim, D-68138 Mannheim, Germany

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Abstract: The structure of the constraint set in the Yang–Mills–Dirac theory in a contractible bounded domain is analysed under the bag boundary conditions. The gauge symmetry group is identified, and it is proved that its action on the phase space is proper and admits slices. The reduced phase space is shown to be the union of symplectic manifolds, each of which corresponds to a definite mode of symmetry breaking.

1. Introduction

In a previous paper we have proved the existence and uniqueness theorems for minimally interacting Yang–Mills and Dirac fields in a bounded contractible domain $M \subset \mathbb{R}^3$, [1]. The aim of this paper is to study the structure of the space of solutions.

Our results were obtained for Cauchy data $\mathbf{A} \in H^2(M)$, $\mathbf{E} \in H^1(M)$, and $\Psi \in H^2(M)$, where $H^k(M)$ is the Sobolev space of fields on M which are square integrable together with their derivatives up to the order k, satisfying the boundary conditions

$$n\mathbf{E} = 0, \quad t\mathbf{B} = 0, \quad in_j\gamma^j\Psi|_{\partial M} = \Psi|_{\partial M}\,, \tag{1.1a}$$

$$n\mathbf{A} = 0, \quad in_j\gamma^j\{\gamma^0(\gamma^k\partial_k + im)\Psi\}|_{\partial M} = \gamma^0(\gamma^k\partial_k + im)\Psi|_{\partial M}\,. \tag{1.1b}$$

Here we use the notation established in [1]. In particular, $n\mathbf{E}$ denotes the normal component of the "electric" part, $t\mathbf{B}$ the tangential component of the "magnetic" part of the field strength on the boundary ∂M of M. Thus, the extended phase space of the theory under consideration is

$$\mathbf{P} = \{(\mathbf{A}, \mathbf{E}, \Psi) \in H^2(M) \times H^1(M) \times H^2(M)| \text{ satisfying } (1.1\text{a,b})\}\,. \tag{1.2}$$

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