Commun. Math. Phys. 176, 1–22 (1996)

Communications in Mathematical Physics

Double Wells: Nevanlinna Analyticity, Distributional Borel Sum and Asymptotics

E. Caliceti[1], **V. Grecchi**[1], **M. Maioli**[2]

[1] Dipartimento di Matematica, Università di Bologna, I-40127 Bologna, Italy
[2] Dipartimento di Matematica, Università di Modena, I-41100 Modena, Italy

Received: 22 November 1994

Abstract: We consider the energy levels of a Stark family, in the parameter j, of quartic double wells with perturbation parameter g:

$$H(g,j) = p^2 + x^2(1-gx)^2 - j\left(gx - \frac{1}{2}\right).$$

For non-even j (and for the symmetric case $j = 0$) we prove analyticity in the full Nevanlinna disk $\Re g^{-2} > R^{-1}$ of the g^2-plane, as predicted by Crutchfield. By means of an approximation we give a heuristic estimate of the asymptotic small g behaviour, showing the relation between the avoided crossings and the failure of the usual perturbation series.

1. Introduction

The eigenvalues of the quartic anharmonic oscillator

$$A(g^2) = p^2 + x^2 + g^2x^4 \tag{1.1}$$

are interesting examples of Borel summability of the Rayleigh Schrödinger perturbation series [Gr-Gr-Si]. The unstable anharmonic oscillator (or "volcano")

$$\tilde{A}(g^2) = p^2 + x^2 - g^2x^4 \tag{1.2}$$

has "resonances" defined as the eigenvalues of the analytic continuations $A[(\pm ig)^2]$ of $A(g^2)$. Such "resonances," as well as the Hydrogen Stark effect resonances, are given by a pair of distributional Borel sums called upper and lower Borel sums (US, LS) [Ca-Gr-Ma1, 2, 4].

The energy levels of the double well Stark family

$$H(g,j) = p^2 + x^2(1-gx)^2 - j\left(gx - \frac{1}{2}\right) \tag{1.3}$$

Partially supported by G.N.F.M. of C.N.R., M.U.R.S.T. and I.N.F.N.