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A Symplectic Structure for Connections on Surfaces with Boundary

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Abstract: For compact surfaces with one boundary component, and semisimple gauge groups, we construct a closed gauge invariant 2-form on the space of flat connections whose boundary holonomy lies in a fixed conjugacy class. This form descends to the moduli space under the action of the full gauge group, and provides an explicit description of a symplectic structure for this moduli space.

1. Introduction

We construct a natural gauge invariant 2-form on the space of all connections over a compact surface with one boundary component, for semisimple gauge groups. When restricted to the space of flat connections whose boundary holonomy lies in a fixed conjugacy class, this 2-form is closed and descends to the moduli space under the action of the full gauge group. We show that for a non-empty open set of conjugacy classes, this gives a symplectic structure on the moduli space.

The moduli space of flat connections on a closed surface has been studied from many viewpoints ([AB, BG, Go, Hi, Hu, HJ, J, JW1,2, Ka, KS1,2, Se2, We, Wi1,2]). A symplectic structure on the moduli space of flat connections with fixed boundary holonomy conjugacy class has been discussed in [We] and is also mentioned in [A]. Several works, including [A, BG, J, Wi1], have investigated the closely related issue of a symplectic structure on the moduli space of flat connections on a surface with marked points.

The present paper uses results from [KS3], where a 2-form was constructed on the moduli space of flat connections on a compact surface with boundary, under the group of based gauge transformations. Ideas and techniques from [KS1] and [KS2] are also used. An underlying theme is the study of certain naturally defined differential forms on the moduli space of flat connections and their counterparts on the infinite dimensional space of all connections.

The paper is organised as follows. In Sect. 2 the general setup is described and the infinite dimensional space $\mathcal{A}$ of all connections is introduced, along with certain (infinite dimensional) subsets $\mathcal{A}^{fl}$, $\mathcal{A}(\Theta)$ and $\mathcal{A}^{fl}(\Theta)$, of interest. In general we use Θ to denote a conjugacy class in the group G, and so we attach the symbol Θ