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The Smooth Cohomology of $N=2$ Supersymmetric Landau–Ginzburg Field Theories

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Abstract: We compute the smooth cohomology (both unrestricted and compactly supported) of the supercharge of an ultraviolet cutoff $N = 2$ supersymmetric Landau–Ginzburg field theory.

I. Introduction

I.A. The two-dimensional $N=2$ supersymmetric Landau–Ginzburg models (also known as Wess–Zumino models) were introduced in the physics literature in the seventies. For early results, see [CGP] and references therein. Constructive field theory aspects of these models were a subject of many investigations, see [JL] and [J] for references. The Landau–Ginzburg models provide useful examples to study complex physical and mathematical phenomena of supersymmetric quantum field theory which are much harder to control in the four dimensional world. Recent revival of interest in the Landau–Ginzburg models stems largely from the fact that they seem to play a role in various "compactification" scenarios of string theory (see e.g. [CV] and references therein).

Supersymmetric quantum field theories provide non-trivial examples of infinite dimensional non-commutative geometries [C]. In particular, supersymmetric field theories with $N=2$ supersymmetries lead naturally to structures which can be regarded as examples of non-commutative Kähler geometry. For the Landau–Ginzburg models, the underlying infinite dimensional geometry is flat. What makes them non-trivial is the non-linear self-interaction term in the Hamiltonian. One of the fundamental difficulties in studying the mathematical structures associated with this model is of technical character: to show that the Hamiltonian is well defined on a dense domain, and that its heat kernel is trace class. This requires a detailed analysis of a suitably regularized form of the Hamiltonian.

I.B. In this paper, we choose a particular regularization, namely the sharp ultraviolet cutoff M. This amounts to suppressing all the modes with $|p| > M$ in the

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