Commun. Math. Phys. 175, 565–606 (1996)

Communications in Mathematical Physics



# The Spectral Gap for Some Spin Chains with Discrete Symmetry Breaking

**Bruno Nachtergaele**

Department of Physics, Princeton University, Princeton, NJ 08544-0708, USA.
E-mail: bxn@math.princeton.edu

Received: 2 November 1994 / in revised form: 9 March 1995

**Abstract:** We prove that for any finite set of generalized valence bond solid (GVBS) states of a quantum spin chain there exists a translation invariant finite-range Hamiltonian for which this set is the set of ground states. This result implies that there are GVBS models with arbitrary broken discrete symmetries that are described as combinations of lattice translations, lattice reflections, and local unitary or anti-unitary transformations. We also show that all GVBS models that satisfy some natural conditions have a spectral gap. The existence of a spectral gap is obtained by applying a simple and quite general strategy for proving lower bounds on the spectral gap of the generator of a classical or quantum spin dynamics. This general scheme is interesting in its own right and therefore, although the basic idea is not new, we present it in a system-independent setting. The results are illustrated with a number of examples.

## Table of Contents