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Supercoherent States, Super-Kähler Geometry and Geometric Quantization

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Abstract: Generalized coherent states provide a means of connecting square integrable representations of a semi-simple Lie group with the symplectic geometry of some of its homogeneous spaces. In the first part of the present work this point of view is extended to the supersymmetric context through the study of the OSp(2/2) coherent states. These are explicitly constructed starting from the known abstract typical and atypical representations of osp(2/2). Their underlying geometries turn out to be those of supersymplectic OSp(2/2)-homogeneous spaces. Moment maps identifying the latter with coadjoint orbits of OSp(2/2) are exhibited via Berezin's symbols. When considered within Rothstein's general paradigm, these results lead to a natural general definition of a super-Kähler supermanifold, the supergeometry of which is determined in terms of the usual geometry of holomorphic Hermitian vector bundles over Kähler manifolds. In particular, the supergeometry of the above orbits is interpreted in terms of the geometry of Einstein–Hermitian vector bundles. In the second part, an extension of the full geometric quantization procedure is applied to the same coadjoint orbits. Thanks to the super-Kähler character of the latter, this procedure leads to explicit super-unitary irreducible representations of osp(2/2) in super-Hilbert spaces of superholomorphic square-integrable sections of prequantum bundles of the Kostant type. This work lays the foundations of a program aimed at classifying Lie supergroups' coadjoint orbits and their associated irreducible representations, ultimately leading to harmonic superanalysis. For this purpose a set of consistent conventions is exhibited.

Contents

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