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The Strong Decay to Equilibrium for the Stochastic Dynamics of Unbounded Spin Systems on a Lattice

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Abstract: Using a method based on the application of hypercontractivity we prove the strong exponential decay to equilibrium for a stochastic dynamics of unbounded spin system on a lattice.

0. Introduction

In recent years essential progress has been made in understanding the ergodicity properties of the Markov semigroups P_t, $t \in \mathbb{R}^+$ defined on the space of continuous functions $\mathscr{C}(\Omega)$, with a configuration space $\Omega \equiv M^\Gamma$, M being a compact metric space and Γ a countable (infinite) set. An important method for the study of these properties was first introduced in [HS]. It involves three elements:

(i) a strong approximation property of the semigroup P_t, $t \in \mathbb{R}^+$ by the semigroups $P_t^{\Lambda,\omega}$ acting (essentially) on $\mathscr{C}(M^\Lambda)$, $\Lambda \subset \Gamma$ finite sets, and fixing a configuration $\omega \in \Omega$ outside Λ,

(ii) the finite volume ultracontractivity property of $P_t^{\Lambda,\omega}$, and

(iii) the uniform in volume Λ and boundary conditions ω hypercontractivity property of the semigroups $P_t^{\Lambda,\omega}$ on the spaces $L_p(E_\Lambda^\omega)$, $p \in (1,\infty)$, with E_Λ^ω being the corresponding invariant probability measures.

The first two properties have been well known for a long time for the situation of compact configuration space. Although the hypercontractivity property of a semigroup, or its equivalent property of corresponding invariant measure called the logarithmic Sobolev inequality (LS), was introduced almost twenty years ago, [G], for many years no nontrivial example involving an infinite dimensional configuration space was known. (For the trivial one corresponding to the Gaussian or some product measures see [G].) This was until a very nice Bakry–Emery criterion (B-E) for the logarithmic Sobolev inequality has been introduced in [BE], for a case of configuration space defined with a (finite dimensional) smooth, connected and compact Riemannian manifold M with positive Ricci curvature (or a case when the Ricci curvature is zero, but involving some special log-concave