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Solitons and Helices: The Search for a Math-Physics Bridge

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Abstract: We present evidence for an undiscovered link between $N = 2$ supersymmetric quantum field theories and the mathematical theory of helices of coherent sheaves. We give a thorough review for nonspecialists of both the mathematics and physics involved, and invite the reader to take up the search for this elusive connection.

1. Introduction

Last year, Kontsevich noticed a similarity between the work of Cecotti and Vafa on classifying two-dimensional $N = 2$ supersymmetric field theories and the work of some algebraic geometers in Moscow [1]. In the independent and seemingly unrelated work of physicists and mathematicians, similar structures emerged. Both had found quasi-unipotent matrices satisfying certain Diophantine conditions, which supported the action of the braid group. Were they the same?

Behind this question lies a potential relationship between disparate fields and the opportunity for string theory and its offshoots to once again bring mathematicians and physicists together. Unfortunately, my search for this bridge was somewhat in vain. I cannot tout complete success; instead I offer an amalgam of evidence and observations supporting this conjecture, along with various approaches used in trying to find this elusive link. These diverse techniques span a breadth of physics and mathematics. This paper is intended to give a thorough treatment while remaining somewhat self-contained, perhaps at the expense of brevity.

The physics is the theory of classifying two-dimensional $N = 2$ supersymmetric field theories [2] and is closely related to topological-anti-topological (tt^*) fusion [3]. The idea for classification was to obtain information about the number of vacua and solitons between them in the infrared limit. Given a massive $N = 2$ theory (we will always consider two-dimensional theories), one can consider the whole renormalization group trajectory – its infrared and ultraviolet limits. In the conformal, or ultraviolet, limit, the (universality class of the) theory can be partially classified

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