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A Contribution of the Trivial Connection to the Jones Polynomial and Witten's Invariant of 3d Manifolds, I

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Abstract: We use a path integral formulation of the Chern–Simons quantum field theory in order to give a simple "semi-rigorous" proof of a recently conjectured limitation on the $1/K$ expansion of the Jones polynomial of a knot and its relation to the Alexander polynomial. A combination of this limitation with the finite version of the Poisson resummation allows us to derive a surgery formula for the contribution of the trivial connection to Witten's invariant of rational homology spheres. The 2-loop part of this formula coincides with Walker's surgery formula for the Casson–Walker invariant. This proves a conjecture that the Casson–Walker invariant is proportional to the 2-loop correction to the trivial connection contribution. A contribution of the trivial connection to Witten's invariant of a manifold with nontrivial rational homology is calculated for the case of Seifert manifolds.

1. Introduction

In his paper [1], Witten defined a topological invariant of a 3d manifold M with an n-component link $\mathscr{L}$ inside it as a partition funciton of a quantum Chern–Simons theory. Let us attach representations $V_{\alpha_i}, 1 \leqq i \leqq n$ of a simple Lie group G to the components of $\mathscr{L}$ (in our notations α_i are the highest weights shifted by $\rho = \frac{1}{2}\sum_{\lambda_i \in \Delta_+} \lambda_i$, Δ_+ is a set of positive roots of G). Then Witten's invariant is equal to the path integral over all guage equivalence classes of G connection on M:

$$Z_{\alpha_1,\ldots,\alpha_n}(M,\mathscr{L};k) = \int[\mathscr{D}A_\mu]\exp\left(\frac{i}{\hbar}S_{CS}\right)\prod_{i=1}^{n}\mathrm{Tr}_{\alpha_i}\,\mathrm{Pexp}\left(\oint_{L_i} A_\mu dx^\mu\right)\,, \tag{1.1}$$

here A_μ is a connection, S_{CS} is its Chern–Simons action,

$$S_{CS} = \frac{1}{2}\,\mathrm{Tr}\,\varepsilon^{\mu\nu\rho}\int_M dx\left(A_\mu\partial_\nu A_\rho + \frac{2}{3}A_\mu A_\nu A_\rho\right)\,, \tag{1.2}$$

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