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Uniform Boundedness of the Solutions for a One-Dimensional Isentropic Model System of Compressible Viscous Gas

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Abstract: This paper studies an initial boundary value problem for a one-dimensional isentropic model system of compressible viscous gas with large external forces, represented by $v_t - u_x = 0$, $u_t + (av^{-\gamma})_x = \mu(u_x/v)_x + f(\int_0^x v dx, t)$, with $(v(x,0), u(x,0)) = (v_0(x), u_0(x))$, $u(0,t) = u(1,t) = 0$. Especially, the uniform boundedness of the solution in time is investigated. It is proved that for arbitrary large initial data and external forces, the problem uniquely has an uniformly bounded, global-in-time solution with also uniformly positive mass density, provided the adiabatic constant $\gamma(> 1)$ is suitably close to 1. The proof is based on L^2-energy estimates and a technique used in [9].

1. Introduction

In this paper we consider the one-dimensional motion of a general viscous isentropic gas in a bounded region, with an external force. In the Lagrangian mass coordinate, such a motion is described by the following system of equations:

$$v_t - u_x = 0 \,, \tag{1.1}$$

$$u_t + p(v)_x = \mu\left(\frac{u_x}{v}\right)_x + f\left(\int_0^x v dx, t\right) \,, \tag{1.2}$$

where v, u, p, μ and f in the equations are the specific volume, the velocity, the pressure, the viscosity coefficient, and the external force of the fluid, respectively. We will assume that the equation of state, i.e., the function p is given by

$$p(v) = av^{-\gamma} \quad (a > 0,\ \gamma > 1 \text{ are the constants}) \,, \tag{1.3}$$

and that the viscosity coefficient is a positive constant. After normalization, we may assume without loss of generality that the fluid occupies the interval $(0, 1)$, whose