Commun. Math. Phys. 175, 177 – 202 (1996)

Communications in
Mathematical
Physics

The Integrable Hierarchy Constructed from a Pair of KdV-Type Hierarchies and its Associated W Algebra

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Received: 16 August 1994 / in revised form: 15 March 1995

Abstract: For any two arbitrary positive integers "n" and "m", using the m^{th} KdV hierarchy and the $(n+m)^{\text{th}}$ KdV hierarchy as building blocks, we are able to construct another integrable hierarchy (referred to as the $(n,m)^{\text{th}}$ KdV hierarchy). The W-algebra associated to the second Hamiltonian structure of the $(n,m)^{\text{th}}$ KdV hierarchy (called $W(n,m)$ algebra) is isomorphic via a Miura map to the direct sum of a W_m-algebra, a W_{n+m}-algebra and an additional $U(1)$ current algebra. In turn, from the latter, we can always construct a representation of a W_∞-algebra.

1. Introduction

Our purpose in this paper is to show how to construct new integrable hierarchies starting from a couple of KdV-type hierarchies plus a $U(1)$ current. Also in order to give the coordinates of our paper with respect to the current literature, let us recall a few fundamental things about KdV hierarchies.

There are two different descriptions of the n^{th} KdV hierarchy. One is based on the so-called pseudodifferential operator analysis (see [1]), in which we start from a differential operator L, called scalar Lax operator,

$$L = \partial^n + \sum_{i=1}^{n-1} u_i \partial^{n-i-1}, \qquad \partial = \frac{\partial}{\partial x}, \tag{1.1}$$

where the u_i's are functions of the "space" coordinate x. Throughout the paper the symbol L will mean (1.1). After introducing the inverse ∂^{-1} of the derivative ∂ (i.e. the formal integration operator),

$$\partial\partial^{-1} = \partial^{-1}\partial = 1,$$

$$\partial^{-1} f(x) = \sum_{l=0}^{\infty} (-1)^l f^{(l)} \partial^{-l-1},$$

we can calculate the fractional powers of L.