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Free q-Schrödinger Equation from Homogeneous Spaces of the 2-dim Euclidean Quantum Group

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Abstract: After a preliminary review of the definition and the general properties of the homogeneous spaces of quantum groups, the quantum hyperboloid qH and the quantum plane qP are determined as homogeneous spaces of $\mathscr{F}_q(E(2))$. The canonical action of $E_q(2)$ is used to define a natural q-analog of the free Schrödinger equation, that is studied in the momentum and angular momentum bases. In the first case the eigenfunctions are factorized in terms of products of two q-exponentials. In the second case we determine the eigenstates of the unitary representation, which, in the qP case, are given in terms of Hahn–Exton functions. Introducing the universal T-matrix for $E_q(2)$ we prove that the Hahn–Exton as well as Jackson q-Bessel functions are also obtained as matrix elements of T, thus giving the correct extension to quantum groups of well known methods in harmonic analysis.

1. Introduction

The fundamental role played by homogeneous spaces in harmonic analysis and in applications to physical theories cannot be overestimated. Apparently different mathematical problems, like the definition of special functions and integral transforms, from the one side, and the classification of elementary Hamiltonian systems by means of coadjoint orbits and their quantization according to the Kirillov theory [1], from the other, find their unifying *leitmotiv* in homogeneous spaces. Also the fundamental wave equations of mathematical physics have their natural origin in the study of homogeneous spaces of groups with kinematical or dynamical meaning, such as the Euclidean or the Poincaré group: more specifically, they are determined by the canonical action of the Casimir of the corresponding Lie algebra on spaces of functions on these homogeneous manifolds.

With the development of the theory of quantum groups and just after the first steps in the study of their structure, it seemed extremely natural to investigate the analogs of homogeneous spaces in this new quantum framework. As the notion of manifold underlying the algebraic structure is obviously lacking, the right approach starts from the injection of the algebra of the quantum functions of the homogeneous space into the algebra of the quantum functions of the group. After the pioneering