

Compensations in Small Divisor Problems

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Abstract: A general direct method, alternative to KAM theory, apt to deal with small divisor problems in the real-analytic category, is presented and tested on several small divisor problems including the construction of maximal quasi-periodic solutions for nearly-integrable non-degenerate Hamiltonian or Lagrangian systems and the construction of lower dimensional resonant tori for nearly-integrable Hamiltonian systems. The method is based on an explicit graph theoretical representation of the formal power series solutions, which allows to prove compensations among the monomials forming such representation.

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1. Introduction

1. Small divisors are ubiquitous in non-linear conservative dynamical systems; they arise, for example, in: conjugacy problems such as linearizations of germs of analytic functions or linearizations of circle maps (see, e.g., [1] and references therein);

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