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Spectra, Eigenvectors and Overlap Functions for Representation Operators of q-Deformed Algebras

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Abstract: Operators of representations corresponding to symmetric elements of the q-deformed algebras $U_q(\mathrm{su}_{1,1}), U_q(\mathrm{so}_{2,1}), U_q(\mathrm{so}_{3,1}), U_q(\mathrm{so}_n)$ and representable by Jacobi matrices are studied. Closures of unbounded symmetric operators of representations of the algebras $U_q(\mathrm{su}_{1,1})$ and $U_q(\mathrm{so}_{2,1})$ are not selfadjoint operators. For representations of the discrete series their deficiency indices are $(1, 1)$. Bounded symmetric operators of these representations are trace class operators or have continuous simple spectra. Eigenvectors of some operators of representations are evaluated explicitly. Coefficients of transition to eigenvectors (overlap coefficients) are given in terms of q-orthogonal polynomials. It is shown how results on eigenvectors and overlap coefficients can be used for obtaining new results in representation theory of q-deformed algebras.

1. Introduction

There is a connection between representations of a semisimple Lie group and representations of its Lie algebra [1]. To noncompact generators I there correspond unbounded operators in infinite dimensional irreducible representations T of a semisimple Lie algebra g. To every such representation T of g there corresponds an irreducible representation T of the Lie group G with the Lie algebra g. Operators of a representation T of G are bounded. If a representation T of G is unitary, then to noncompact generators I from g, multipled by $\mathrm{i} = \sqrt{-1}$, there correspond symmetric operators on a Hilbert space. Unitarity of a representation T of G means that closures of these symmetric operators are selfadjoint operators. Properties of self-adjointness for operators corresponding to symmetric elements of the universal enveloping algebra $U(g)$ of g are also well known (see, [1], Chapter 11).

The corresponding theory is absent for infinite dimensional representations of quantum algebras. Moreover, simple examples show that the situation for quantum algebras is unlike that which we have in the classical case.

Quantum algebras are q-deformed universal enveloping algebras $U_q(g)$ corresponding to simple Lie algebras (we do not consider here quantum algebras corresponding to affine Lie algebras). Such q-deformations are constructed for all