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The Euler–Poincaré Equations and Double Bracket Dissipation

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Abstract: This paper studies the perturbation of a Lie–Poisson (or, equivalently an Euler–Poincaré) system by a special dissipation term that has Brockett's double bracket form. We show that a formally unstable equilibrium of the unperturbed system becomes a spectrally and hence nonlinearly unstable equilibrium after the perturbation is added. We also investigate the geometry of this dissipation mechanism and its relation to Rayleigh dissipation functions. This work complements our earlier work (Bloch, Krishnaprasad, Marsden and Ratiu [1991, 1994]) in which we studied the corresponding problem for systems with symmetry with the dissipation added to the internal variables; here it is added directly to the group or Lie algebra variables. The mechanisms discussed here include a number of interesting examples of physical interest such as the Landau–Lifschitz equations for ferromagnetism, certain models for dissipative rigid body dynamics and geophysical fluids, and certain relative equilibria in plasma physics and stellar dynamics.

1. Introduction

The purpose of this paper is to study the phenomenon of dissipation induced instabilities for Euler–Poincaré systems on Lie algebras or equivalently, for Lie–Poisson systems on the duals of Lie algebras. Lie–Poisson systems on the duals of Lie algebras $\mathfrak{g}^*$ are obtained by reduction from invariant Hamiltonian systems on cotangent

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