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Scaling Behaviour in the Bose Gas

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Abstract: The scaling behaviour of fluctuations of the Bose fields $\Phi(f)$ in the ergodic infinite volume equilibrium states of a d-dimensional Bose gas at temperature T and density $\overline{\rho}$, can be classified in terms of the testfunctions f. In the low density regime, the space of testfunctions splits up in two subspaces, leading to two different types of non-commuting macroscopic field fluctuation observables. Testfunctions f with Fourier transform $\hat{f}(0) \neq 0$ yield normal fluctuation observables. The local fluctuations of the field operators $\Phi(f)$ must be scaled subnormally (i.e. with a negative scaling index) if the testfunction f has $\hat{f}(0) = 0$. The macroscopic fluctuations of these fields can then again be described by a Bose field. The situation changes when the density of the gas exceeds the critical density. The field operators which have normal fluctuations in the low density regime need to be scaled abnormally in the high density regime, yielding classical macroscopic fluctuation observables. Another difference with the low density regime is that the space of testfunctions with $\hat{f}(0) = 0$ splits up in two subspaces when the critical density is reached: for a first subspace the algebraic character of the macroscopic field fluctuation observables is also classical because it is necessary to scale the fluctuations of the field operators normally, while for the remaining subclass, the same negative scaling index is required as in the low density regime and hence also the algebraic character of these macroscopic fluctuations is again CCR.

1. Introduction

In the temperature–density $(T, \overline{\rho})$ phase diagram of a free Bose gas, one distinguishes a low density regime and a high density regime separated by a line of critical densities $\overline{\rho} = \rho_c(T, d)$. In the low density regime and on the critical line there is a unique gauge invariant ergodic infinite volume equilibrium state for each inverse temperature β and density $\overline{\rho}$.

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