Commun. Math. Phys. 174, 605–633 (1996)

Communications in
Mathematical
Physics

Characterizing Invariants for Local Extensions of Current Algebras

Karl-Henning Rehren[1], Yassen S. Stanev[2,3], Ivan T. Todorov[2,3]

[1] II. Institut für Theoretische Physik, Universität Hamburg, D-22761 Hamburg, Germany
[2] Erwin Schrödinger International Institute for Mathematical Physics, A-1090 Wien, Austria
[3] Institute for Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, BG-1784 Sofia, Bulgaria*

Received: 3 October 1994 / in revised form: 20 February 1995

Abstract: Pairs $\mathscr{A} \subset \mathscr{B}$ of local quantum field theories are studied, where $\mathscr{A}$ is a chiral conformal quantum field theory and $\mathscr{B}$ is a local extension, either chiral or two-dimensional. The local correlation functions of fields from $\mathscr{B}$ have an expansion with respect to $\mathscr{A}$ into conformal blocks, which are non-local in general. Two methods of computing characteristic invariant ratios of structure constants in these expansions are compared: (a) by constructing the monodromy representation of the braid group in the space of solutions of the Knizhnik–Zamolodchikov differential equation, and (b) by an analysis of the local subfactors associated with the extension with methods from operator algebra (Jones theory) and algebraic quantum field theory. Both approaches apply also to the reverse problem: the characterization and (in principle) classification of local extensions of a given theory.

1. Introduction

The relevance of V. Jones' theory of (von Neumann) subfactors [1] for 2-dimensional (2D) models of critical behaviour was first recognized in the work of V. Pasquier on lattice models labelled by Dynkin diagrams [2]. A spectacular by-product of this realization was the ensuing *ADE* classification of $su(2)$ current algebra models and minimal conformal theories [3]. The above parallel was understood within the Haag–Kastler algebraic approach to local quantum field theory [4] in terms of the Doplicher–Haag–Roberts (DHR) theory of superselection sectors and particle statistics [5] applied to chiral algebras [6, 7], and provided an explanation for the Jones index as a measure for the violation of Haag duality (maximality of local observables) in a given representation, and relating it numerically to the statistical dimension [8].

In the cited work on subfactors in quantum field theory, the emphasis for the use of the theory of subfactors was its application to individual superselection sectors of a given local theory, and the derivation of invariant "charge quantum numbers" such as statistical dimensions and Markov traces. In contrast, here we shall consider a

* permanent address