Commun. Math. Phys. 174, 477–507 (1996)

Communications in Mathematical Physics

Quantum Spin Chains with Quantum Group Symmetry

M. Fannes[1,2], B. Nachtergaele[3,4], R.F. Werner[5]

[1] Inst. Theor. Fysica, Universiteit Leuven, B-3001 Heverlee, Belgium
e-mail: mark.fannes@fys.kuleuven.ac.be
[2] Onderzoeksleider, N.F.W.O. Belgium
[3] Dept. of Physics, Princeton University, Princeton, NJ 08544-0708, USA
e-mail: bxn@math.princeton.edu
[4] Supported in part by NSF Grant # PHY90-19433 A02
[5] Fachbereich Physik, Universität Osnabrück, D-49069 Osnabrück, Germany
e-mail: reinwer@dosuni1.rz.uni-osnabrueck.de

Received: 6 June 1994 / in revised form: 1 March 1995

Abstract: We consider actions of quantum groups on lattice spin systems. We show that if an action of a quantum group respects the local structure of a lattice system, it has to be an ordinary group. Even allowing weakly delocalized (quasi-local) tails of the action, we find that there are no actions of a properly quantum group commuting with lattice translations. The non-locality arises from the ordering of factors in the quantum group C^*-algebra, and can be made one-sided, thus allowing semi-local actions on a half chain. Under such actions, localized quantum group invariant elements remain localized. Hence the notion of interactions invariant under the quantum group and also under translations, recently studied by many authors, makes sense even though there is no global action of the quantum group. We consider a class of such quantum group invariant interactions with the property that there is a unique translation invariant ground state. Under weak locality assumptions, its GNS representation carries no unitary representation of the quantum group.

1. Introduction

Symmetry has always played an important role in theoretical physics in helping to reduce a problem with many variables to a more tractable size. In statistical mechanics we have infinitely many degrees of freedom to deal with, so often the symmetry, while helpful, is not sufficient to solve the problem, unless we have "infinitely many symmetries." One example is the theory of mean-field lattice systems, where the inherent permutation symmetry is sufficient to reduce the computation of the limit free energy density, of the possible limit states [FSV, RW], and of the limit dynamics [DW] to corresponding problems in the algebra for a single spin.

Another example, which has been studied intensively by many authors recently [Bab, BMNR, DC, KS, MN, GS], is the class of models which can be solved exactly (though not always rigorously) by means of the Bethe Ansatz. The basis of this method is to diagonalize the Hamiltonian along with an infinite set of constants of motion. In some cases the occurrence of this infinite set of constants of motion