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Wave Equations on q-Minkowski Space

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Abstract: We give a systematic account of a "component approach" to the algebra of forms on q-Minkowski space, introducing the corresponding exterior derivative, Hodge star operator, coderivative, Laplace-Beltrami operator and Lie-derivative. Using this (braided) differential geometry, we then give a detailed exposition of the q-d'Alembert and q-Maxwell equation and discuss some of their non-trivial properties, such as for instance, plane wave solutions. For the q-Maxwell field, we also give a q-spinor analysis of the q-field strength tensor.

1. Introduction

This paper develops some elements of braided differential geometry on quantum Minkowski space and uses these new tools to define and analyse the two simplest wave equations on this non-commutative spacetime, namely the q-d'Alembert and the q-Maxwell equation. In order to distinguish our approach from other related work it might be useful to emphasize that we are constructing generalisations of *classical* wave equations in position space and not a deformation of quantum theory, as for instance in [13], where wave equations in momentum space were constructed by using irreducible representations of the q-Poincaré group. At present, there is no q-Fourier transform in this case, and it does not seem to be possible to compare the results of the two approaches.

In our exposition of braided differential geometry we present forms in a slightly different way than in some earlier papers by other authors. We use what one might call a *component approach* to forms, but will show in Proposition 2.7 that the two possible approaches are equivalent. This different approach to forms has the consequence that the q-exterior derivative d is constructed in terms of braided differential operators ∂^a and not vice versa, as for instance in [12]. The additional ingredient needed for this construction is a q-Lorentz covariant antisymmetrisation operation, which we introduce. In a similar fashion, we also define the q-Hodge star operator, q-coderivative, q-Lie derivative, and q-Laplace-Beltrami operator. The advantage of the component approach is that for instance the q-electromagnetic field is given terms of components and admits a very simple $SL_q(2,\mathbb{C})$-spinor decomposition into