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Singular Vectors of the Virasoro Algebra in Terms of Jack Symmetric Polynomials

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Abstract: We present an explicit formula of the Virasoro singular vectors in terms of Jack symmetric polynomials. The parameter t in the Virasoro central charge $c = 13 - 6(t + 1/t)$ is just identified with the deformation parameter α of Jack symmetric polynomials $J_\lambda(\alpha)$. As a by-product, we obtain an integral representation of Jack symmetric polynomials indexed by the rectangular Young diagrams.

1. Introduction

The Virasoro algebra is an infinite dimensional Lie algebra defined by the generators $l_m (m \in \mathbb{Z})$ and c with the relations

$$[l_n, l_m] = (n - m)l_{m+n} + \frac{c}{12}(n^3 - n)\delta_{m+n,0}$$

for $m, n \in \mathbb{Z}$ and $[c, l_m] = 0$ for $m \in \mathbb{Z}$. Among many mathematicians and physicists to investigate the representation theory of the Virasoro algebra (see [2, 3] for instance), Tsuchiya and Kanie classified the singular vectors of the Fock space representations in [8], where they constructed the integral representation of the singular vectors. But they missed obtaining explicit forms of such singular vectors. The present article will be devoted to this point: Namely, we express the singular vectors in terms of some symmetric polynomials, called Jack symmetric polynomials.

Jack symmetric polynomials were first introduced by Jack in 1970 to evaluate some definite integral. Subsequently, the systematic study was begun by Macdonald, who considered even more general cases: Macdonald's symmetric polynomials [6].

In this paper we show that the singular vectors of the Virasoro algebra are given by Jack symmetric polynomials indexed by the rectangular Young diagrams.

We believe that such realization of Jack symmetric polynomials would be helpful in future research both of the representation theory of infinite dimensional algebras and of hypergeometric type functions in several variables.

Some basic notations about the symmetric polynomials, with definition of Jack symmetric polynomials, are recalled in Sect. 2. We give the main result in Sect. 3,