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Transparent Potentials at Fixed Energy in Dimension Two. Fixed-Energy Dispersion Relations for the Fast Decaying Potentials

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Abstract: For the two-dimensional Schrödinger equation

$$[-\Delta + v(x)]\psi = E\psi, \quad x \in \mathbb{R}^2, \quad E = E_{\text{fixed}} > 0 \quad (*)$$

at a fixed positive energy with a fast decaying at infinity potential $v(x)$ dispersion relations on the scattering data are given. Under "small norm" assumption using these dispersion relations we give (without a complete proof of sufficiency) a characterization of scattering data for the potentials from the Schwartz class $S = C_{\infty}^{(\infty)}(\mathbb{R}^2)$. For the potentials with zero scattering amplitude at a fixed energy E_{fixed} (transparent potentials) we give a complete proof of this characterization. As a consequence we construct a family (parametrized by a function of one variable) of two-dimensional spherically-symmetric real potentials from the Schwartz class S transparent at a given energy. For the two-dimensional case (without assumption that the potential is small) we show that there are no nonzero real exponentially decreasing, at infinity, potentials transparent at a fixed energy. For any dimension greater or equal to 1 we prove that there are no nonzero real potentials with zero forward scattering amplitude at an energy interval. We show that KdV-type equations in dimension 2+1 related with the scattering problem $(*)$ (the Novikov–Veselov equations) do not preserve, in general, these dispersion relations starting from the second one. As a corollary these equations do not preserve, in general, the decay rate faster than $|x|^{-3}$ for initial data from the Schwartz class.

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