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Zariski Closure of Subgroups of the Symplectic Group and Lyapunov Exponents of the Schrödinger Operator on the Strip

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Abstract: We consider the Schrödinger equation with a random potential

$$-y_{n+1} + (Q_n - EI)y_n - y_{n-1} = 0\,,$$

where Q_n is a sequence of independent identically distributed random symmetric $m \times m$-matrices with real valued elements, $y_n \in \mathbb{R}^m$, $-\infty < n < \infty$, E is the real parameter, and I is the identity matrix. We show that if the smallest Jordan algebra of matrices containing the support of the distribution of matrices Q_n coincides with Jordan algebra of all (real-valued) symmetric matrices then for all but (maybe) a finite number of values of E all the Lyapunov exponents of our Schrödinger equation are different (and thus the spectrum of the corresponding Schrödinger operator is pure point).

General Remarks and Main Notations

The study of spectral properties of random Schrödinger operators on a strip (the well-known Anderson model [A]) was initiated in [G] in 1980. At that time, the papers [GM1, GM2] did not yet exist and the case of potentials with singular distributions (singular potentials) couldn't be solved by means of the techniques used in [G]. That is one of the reasons why localization has been proved there for operators with potentials having continuous density distribution. One more reason is worth mentioning. In [GMo, GMoP] localization has been understood for the first time and for the strictly one-dimensional case. The main goal of [G] was to go beyond the one-dimensional Anderson model and because of that the questions concerned with conditions of existence of any kind of densities were considered to be of much smaller importance compared with those related to the understanding of the mechanism of localization phenomena.

In [La1, La2] localization and exponential decay of the corresponding eigenfunctions has been established under similar conditions. A better understanding of the same phenomena was the main goal also for these works and the techniques used in [La1, La2] did not solve the case of singular potentials.