

# On Nonlinear Stability of General Undercompressive Viscous Shock Waves

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**Abstract:** We study the nonlinear stability of general undercompressive viscous shock waves. Previously, the authors showed stability in a special case when the shock phase shift can be determined a priori from the total mass of the perturbation, using new pointwise methods. By examining time invariants associated with the linearized equations, we can now overcome a new difficulty in the general case, namely, nonlinear movement of the shock. We introduce a coordinate transformation suitable to treat this new aspect, and demonstrate our method by analyzing a model system of generic type. We obtain sharp pointwise bounds and  $L^p$  behavior of the solution for all  $p, 1 \leq p \leq \infty$ .

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