

# A Special Class of Stationary Flows for Two-Dimensional Euler Equations: A Statistical Mechanics Description. Part II

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**Abstract:** We continue and conclude our analysis started in Part I (see [CLMP]) by discussing the microcanonical Gibbs measure associated to a N-vortex system in a bounded domain. We investigate the Mean-Field limit for such a system and study the corresponding Microcanonical Variational Principle for the Mean-Field equation. We discuss and achieve the equivalence of the ensembles for domains in which we have the concentration at  $\beta \rightarrow (-8\pi)^+$  in the canonical framework. In this case we have the uniqueness of the solutions of the Mean-Field equation. For the other kind of domains, for large values of the energy, there is no equivalence, the entropy is not a concave function of the energy, and the Mean-field equation has more than one solution. In both situations, we have concentration when the energy diverges. The Microcanonical Mean Field Limit for the N-vortex system is proven in the case of equivalence of ensembles.

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