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Singular Continuous Spectrum for Palindromic Schrödinger Operators

A. Hof[1], O. Knill[2], B. Simon[2]

[1] Department of Mathematics and Statistics, McMaster University, Hamilton, Ontario, Canada L8S 4K1. Work partially supported by NSERC.
[2] Division of Physics, Mathematics and Astronomy, California Institute of Technology, 253-37, Pasadena, CA, 91125, USA. This material is based upon work supported by the National Science Foundation under Grant No. DMS-9101715. The Government has certain rights in this material.

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Abstract: We give new examples of discrete Schrödinger operators with potentials taking finitely many values that have purely singular continuous spectrum. If the hull X of the potential is strictly ergodic, then the existence of just one potential x in X for which the operator has no eigenvalues implies that there is a generic set in X for which the operator has purely singular continuous spectrum. A sufficient condition for the existence of such an x is that there is a $z \in X$ that contains arbitrarily long palindromes. Thus we can define a large class of primitive substitutions for which the operators are purely singularly continuous for a generic subset in X. The class includes well-known substitutions like Fibonacci, Thue-Morse, Period Doubling, binary non-Pisot and ternary non-Pisot. We also show that the operator has no absolutely continuous spectrum for all $x \in X$ if X derives from a primitive substitution. For potentials defined by circle maps, $x_n = 1_J(\theta_0 + n\alpha)$, we show that the operator has purely singular continuous spectrum for a generic subset in X for all irrational α and every half-open interval J.

1. Introduction

Discrete Schrödinger operators with potentials taking values in a finite set $A \subset \mathbb{R}$ have interesting spectral properties. Topological spaces of such operators are obtained by choosing a compact shift-invariant subset X of the compact metric space $A^{\mathbb{Z}}$. If T denotes the left shift on X, the dynamical system (X, T) is called a subshift. Every point $x \in X$ defines an operator on $l^2(\mathbb{Z})$ by

$$(L(x)u)_n = u_{n+1} + u_{n-1} + x_n u_n \ .$$

In $X = A^{\mathbb{Z}}$, any spectral type can occur: the dense set of periodic operators in $A^{\mathbb{Z}}$ have purely absolutely continuous spectrum; for almost all x with respect to any non-trivial product measure on $X = A^{\mathbb{Z}}$ the spectrum is pure point [7]. Hence, by the wonderland theorem [24], there exists a generic set in $A^{\mathbb{Z}}$ for which $L(x)$ has purely singular continuous spectrum. The shift on $A^{\mathbb{Z}}$ has many invariant measures and there are many orbits which are not dense. It is therefore convenient to consider the case of a compact shift-invariant $X \subset A^{\mathbb{Z}}$ that is minimal (i.e., the set of