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Local BRST Cohomology in the Antifield Formalism: I. General Theorems

Glenn Barnich[1,⋆], Friedemann Brandt[2,⋆⋆], Marc Henneaux[1,⋆⋆⋆]

[1] Faculté des Sciences, Université Libre de Bruxelles, Campus Plaine C.P. 231, B-1050 Bruxelles, Belgium

[2] NIKHEF-H, Postbus 41882, NL-1009 DB Amsterdam, The Netherlands

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Abstract: We establish general theorems on the cohomology $H^*(s|d)$ of the BRST differential modulo the spacetime exterior derivative, acting in the algebra of local p-forms depending on the fields and the antifields (= sources for the BRST variations). It is shown that $H^{-k}(s|d)$ is isomorphic to $H_k(\delta|d)$ in negative ghost degree $-k$ $(k > 0)$, where δ is the Koszul–Tate differential associated with the stationary surface. The cohomology group $H_1(\delta|d)$ in form degree n is proved to be isomorphic to the space of constants of the motion, thereby providing a cohomological reformulation of Noether's theorem. More generally, the group $H_k(\delta|d)$ in form degree n is isomorphic to the space of $n-k$ forms that are closed when the equations of motion hold. The groups $H_k(\delta|d)(k > 2)$ are shown to vanish for standard irreducible gauge theories. The group $H_2(\delta|d)$ is then calculated explicitly for electromagnetism, Yang–Mills models and Einstein gravity. The invariance of the groups $H^k(s|d)$ under the introduction of non-minimal variables and of auxiliary fields is also demonstrated. In a companion paper, the general formalism is applied to the calculation of $H^k(s|d)$ in Yang–Mills theory, which is carried out in detail for an arbitrary compact gauge group.

1. Introduction

A major development of field theory in the eighties has been the construction of the antifield-antibracket formalism [1]. This formalism finds its roots in earlier work on the renormalization of Yang–Mills models [2, 3, 4] and quantization of supergravity [5], and enables one to formulate the quantum rules (path integral, Feynman diagrams) for an arbitrary gauge theory in a manner that maintains manifest spacetime covariance.

⋆ Aspirant au Fonds National de la Recherche Scientifique (Belgium)
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⋆⋆⋆ Also at Centro de Estudios Científicos de Santiago, Chile