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Modular Invariance and Characteristic Numbers

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Abstract: We prove that a general miraculous cancellation formula, the divisibility of certain characteristic numbers, and some other topological results related to the generalized Rochlin invariant, the η-invariant and the holonomies of certain determinant line bundles, are consequences of the modular invariance of elliptic operators on loop space.

1. Motivations

In [AW], a gravitational anomaly cancellation formula, which they called the miraculous cancellation formula, was derived from very non-trivial computations. See also [GS] and [GSW], pp. 347–361. This is essentially a formula relating the L-class to the $\hat{A}$-class and a twisted $\hat{A}$-class of a 12-dimensional manifold. More precisely, let M be a smooth manifold of dimension 12, then this miraculous cancellation formula is

$$L(M) = 8\hat{A}(M,T) - 32\hat{A}(M)\,,$$

where $T = TM$ denotes the tangent bundle of M and the equality holds at the top degree of each differential form. Here recall that, if we use $\{\pm x_j\}$ to denote the formal Chern roots of $TM \otimes \mathbb{C}$, then

$$L(M) = \prod_j \frac{x_j}{\tanh x_j/2}, \qquad \hat{A}(M) = \prod_j \frac{x_j/2}{\sinh x_j/2},$$

and

$$\hat{A}(M,T) = \hat{A}(M)\,\mathrm{ch}\,T \text{ with } \mathrm{ch}\,T = \sum_j e^{x_j} + e^{-x_j}\,.$$

Using a computer, Ochanine obtained the expressions of the top degree terms of $\hat{A}(M)$ and $L(M)$ in terms of the Pontryagin classes $\{p_j\}$ of M, also see [AW] or [GSW]:

$$\hat{A}(M) = -\frac{31}{967680}p_1^3 + \frac{11}{241920}p_1p_2 - \frac{1}{60480}p_3\,,$$