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On the Global Existence of Bohmian Mechanics

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Abstract: We show that the particle motion in Bohmian mechanics, given by the solution of an ordinary differential equation, exists globally: For a large class of potentials the singularities of the velocity field and infinity will not be reached in finite time for typical initial values. A substantial part of the analysis is based on the probabilistic significance of the quantum flux. We elucidate the connection between the conditions necessary for global existence and the self-adjointness of the Schrödinger Hamiltonian.

1. Introduction

Bohmian mechanics [7, 8, 4, 13, 14, 17] is a Galilean invariant theory for the motion of point particles. Consider a system of N particles with masses $m_1, \ldots, m_N$ and potential $V = V(\mathbf{Q}_1, \ldots, \mathbf{Q}_N)$, where $\mathbf{Q}_k \in R^\nu$ denotes the position of the k^{th} particle. The relevant configuration space is an open subset of $\nu N = d$-dimensional space R^d, for example the complement of the set of singularities of V, and shall be denoted by Ω. The state of the N-particle system is given by the configuration $Q = (\mathbf{Q}_1, \ldots, \mathbf{Q}_N) \in \Omega$ and the Schrödinger wave function ψ on configuration space Ω. On the subset of Ω where the wave function $\psi \neq 0$ and is differentiable, it generates a velocity field $v^\psi = (\mathbf{v}_1^\psi, \ldots, \mathbf{v}_N^\psi)$,

$$\mathbf{v}_k^\psi(q) = \frac{\hbar}{m_k} \mathrm{Im} \frac{\nabla_k \psi(q)}{\psi(q)}\,, \tag{1}$$

the integral curves of which are the trajectories of the particles. Thus the time evolution of the state (Q_t, ψ_t) is given by a first-order ordinary differential equation for the configuration Q_t,

$$\frac{dQ_t}{dt} = v^{\psi_t}(Q_t)\,, \tag{2}$$

and Schrödinger's equation for the wave function ψ_t,

$$i\hbar \frac{\partial \psi_t(q)}{\partial t} = \left(-\sum_{k=1}^{N} \frac{\hbar^2}{2m_k} \Delta_k + V(q) \right) \psi_t(q)\,, \tag{3}$$