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Mirror Manifolds in Higher Dimension

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Abstract: We describe mirror manifolds in dimensions different from the familiar case of complex threefolds. We isolate certain simplifying features present only in dimension three, and supply alternative methods that do not rely on these special characteristics and hence can be generalized to other dimensions. Although the moduli spaces for Calabi–Yau d-folds are not "special Kähler manifolds" when $d > 3$, they still have a restricted geometry, and we indicate the new geometrical structures which arise. We formulate and apply procedures which allow for the construction of mirror maps and the calculation of order-by-order instanton corrections to Yukawa couplings. Mathematically, these corrections are expected to correspond to calculating Chern classes of various parameter spaces (Hilbert schemes) for rational curves on Calabi–Yau manifolds. Our mirror-aided calculations agree with those Chern class calculations in the limited number of cases for which the latter can be carried out with current mathematical tools. Finally, we make explicit some striking relations between instanton corrections for various Yukawa couplings, derived from the associativity of the operator product algebra.

1. Introduction

Calabi–Yau threefolds were originally introduced into string theory to provide six compact spatial dimensions which complement four Minkowski spacetime directions to yield a consistent ten dimensional background for string propagation. From a more general perspective, Calabi–Yau threefolds can be target spaces for two dimensional supersymmetric ($N = 2$) conformally invariant nonlinear sigma models with $c = 9$ – this number arising from three times the complex dimension of the target space. Such superconformal field theories have interesting applications to string backgrounds and critical systems, and have led to some striking predictions in mathematical physics. In the latter category, the recent conjectures [1], evidence from numerical studies [2], explicit construction [3], and applications [4, 5] of mirror symmetry are indications of a deep mathematical structure that, at present, is best understood from the physical viewpoint. We take that viewpoint throughout this paper.