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The Effective Gauge Field Action of a System of Non-Relativistic Electrons

J. Fröhlich[1], **R. Götschmann**[1,3], **P.A. Marchetti**[2]

[1] Theoretical Physics, ETH-Hönggerberg, CH-8093 Zürich, Switzerland
[2] Dipartimento di Fisica, Università di Padova and I.N.F.N. - Sezione di Padova, I-35131 Padova, Italy
[3] Institut de Physique Théorique, Université de Fribourg, CH-1700 Fribourg, Switzerland

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Abstract: We consider a system of free, non-relativistic electrons at zero temperature and positive density, coupled to an arbitrary, external electromagnetic vector potential, A. By integrating out the electron degrees of freedom we obtain the effective action for A. We show that, in the scaling limit, this effective action is quadratic in A and can be viewed as an integral over the Fermi sphere of effective actions of $(1+1)$-dimensional, chiral Schwinger models. We use this result to elucidate Luther–Haldane bosonization of systems of non-relativistic electrons. We also study systems of weakly coupled interacting electrons for which the BCS channel is turned off. Using the quadratic dependence of the effective action on A, we show that, in the scaling limit, the RPA yields the dominant contribution.

0. Introduction

Non-relativistic electrons are described by two-component Pauli spinors $\psi(x) = \binom{\psi_\uparrow}{\psi_\downarrow}(x)$. The invariance of the system under global phase transformations of the form $\psi(x) \to \psi'(x) = e^{-i\chi}\psi(x)$, with $\chi \in \mathbf{R}$, turns to a local symmetry, with $\chi(x)$ a real-valued function, in the presence of (real-valued) gauge fields $A_\rho(x)$, $\rho = 0, 1, \ldots, d$, with the transformation property $A_\rho(x) \to A_\rho(x) + \partial_\rho \chi(x)$. The gauge fields are coupled to the matter system by replacing derivatives ∂_ρ by covariant ones, $D_\rho(A) = \partial_\rho + iA_\rho$. Electromagnetism provides a realization of this gauge symmetry. The electromagnetic potential A_ρ couples to the charge current of the electron system.

We analyse the coupling of an external gauge field A to a system of non-interacting electrons. In this paper, we neglect the magnetic moment of the electron; the coupling of the electromagnetic field to the spin currents is a higher relativistic correction (cf. [1]). For this system, we show that the leading term in the effective gauge field action in a regime of large distance scales and low frequencies (the so-called scaling limit) is quadratic in the gauge field. This follows from the observation that, in the scaling limit, the effective action, as a functional of the external