Commun. Math. Phys. 173, 401 – 415 (1995)

Communications in
Mathematical
Physics

Existence of Resonances in Three Dimensions

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Received: 26 September 1994/in revised form: 23 December 1994

Abstract: If P is an elliptic self-adjoint perturbation of the Laplacian Δ on $\mathbb{R}^3$, and the coefficients of $P - \Delta$ decay super-exponentially, then we show that P has infinitely many resonances. The resonances are defined here as the poles of the meromorphic continuation of $(P - \lambda^2)^{-1}$.

1. Introduction and Statement of Results

The purpose of this note is to show the existence of infinitely many resonances for super-exponentially decaying elliptic self-adjoint perturbations of the Laplacian in $\mathbb{R}^3$.

The key component is the extension of Melrose's Poisson formula to that class of perturbations:

$$\operatorname{tr}(U(t) - U_0(t)) = \sum_{\text{resonances}} e^{-i\lambda_j t}, \quad t > 0, \tag{1.1}$$

where $U(t)$ denotes the perturbed wave group and $U_0(t)$ the free one and where we included square roots of eigenvalues among resonances. The formula (1.1) was proved for compactly supported potential and obstacle scattering by Melrose [9, 10] and was then extended to more general compactly supported perturbations in [18]. It was used by Sjöstrand and the second author [17, 18] and then by Farhy [4] and Vodev [22] to obtain lower bounds on the number of scattering poles in a variety of situations, always using the strong singularities of the wave group. It is obvious however that non-vanishing of the wave trace for $t > 0$ already guarantees the existence of resonances or eigenvalues. That non-vanishing can be immediately inferred from the non-vanishing of the higher heat/wave coefficients, that it is those which do not correspond to δ-functions at $t = 0$. As was pointed out to us by Melrose, the vanishing of the zeroth coefficient in odd dimensions implies that the number of resonances is infinite (see Proposition 4.3 of [12] for consequences of that for scattering by compactly supported potentials in odd dimensions) but unlike in [17, 18, 4, 22] this does not seem to lead to quantitative information.